# Reply by Author to A. Wortman

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WE thank Dr. Wortman[1] for his comments and for presenting his numerical calculations that substantiate the asymptotic results presented in Ref. 2. It should be pointed out that neither Eq. (2) nor Eq. (6) was obtained by curve-fitting. They are asymptotic expressions for small $C_M$ and the reader is referred to Ref. 3 for details.

The predicted linearity of the expression for the Stanton number is verified by Fig. 1 of Ref. 1 and, in fact, extends well beyond the limits set in Ref. 2. The consistency of the sign convention apparently remains subjective.

The change in boundary-layer thickness was only one of the variables used to discuss the results for the heat transfer. Figure 1 in Ref. 2 shows the change in $T_{aw}$ with $C_M$ and a discussion of the role of the adiabatic wall temperature in the behavior of the Stanton number is given on p. 739 of Ref. 2.

### References

[1] Wortman, A., "Comments on the 'Increase of Boundary-Layer Heat Transfer by Mass Injection'," *AIAA Journal*, Vol. 12, No. 4, April 1974, p. 573.

[2] Gersten, K. and Gross, J. F., "Increase of Boundary-Layer Heat Transfer by Mass Injection," *AIAA Journal*, Vol. 11, No. 5, May 1973, pp. 738–739.

[3] Gersten, K. and Gross, J. F., "The Second-Order Boundary-Layer Along a Circular Cylinder in Supersonic Flow," *International Journal of Heat and Mass Transfer*, Vol. 15, Dec. 1973, pp. 2241–2260.

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# Comment on "Mean Velocity Profile of a Thick Turbulent Boundary Layer along a Circular Cylinder"

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THE author has seen with much interest the recent report by Chase[1] and the comments of Bradshaw and Patel[2] on it. Bradshaw and Patel cite the example of the turbulent boundary layer flow with transpiration as an example of the failure of the hypothesis first proposed in Ref. 3. This hypothesis, which essentially implies that the same scaling laws that apply to the sublayer should persist into the region of the law of the wall, has not been properly tested by Bradshaw and Patel which has led them to erroneous conclusions. The purpose of this Comment is to show that the hypothesis of Ref. 3 in fact applies equally well to the case of transpiration also and to offer further comments on some of the other points made by the above authors.

The sublayer equation for transpiration

$$(u/v_*) = (v_*/V_w)[\exp(V_w y/v) - 1] \tag{1}$$

is recast into the following form:

$$(v_*/V_w)\ln[1 + (V_w u/v_*^2)] = yv_*/v \tag{2}$$

The form Eq. (2) is suggested by, for example reference to Eq. (4.9) of Townsend.[4] Logically, the law of the wall is then considered in the form

$$(v_*/V_w)\ln[1 + (V_w u/v_*^2)] = K\ln(yv_*/v) + C \tag{3}$$

A check of Eq. (3) with the form of the bilogarithmic law shows that Eq. (3) has all the properties of the bilogarithmic law. Unlike the bilogarithmic law, Eq. (3) tends to the ordinary logarithmic law for small values of $(V_w u/v_*)$. In a manner similar to the bilogarithmic law and Townsend's[4] Eq. (4.9), it also leaves open the nature of the flow at large suction when the term within the brackets in the left-hand side becomes negative. In view of the well known variation of the von Kármán constant with suction and injection, it is to be expected that $K$ will be a function of the flow through the wall. With $C = 6.0$ and $K = (1/2)$ times the normal value (not unusual), a comparison of the values of $(u/v_*)$ from Eq. (3) and the bilogarithmic law for the example given by Bradshaw and Patel[2] is given in Table 1, which shows that the results from the two formulations match within 4%.

**Table 1 Comparison of $u/v_*$ from bilogarithmic law and Eq. (3)**

| $yu_*/v$ | $(u/v_*)$ bilogarithmic law | $(u/v_*)$ Eq. (3) |
|---|---|---|
| 100 | 22.8 | 22.7 |
| 1000 | 32.4 | 33.7 |

In view of this further evidence, it is clearly not a matter of luck that the hypothesis of Ref. 3 works. It is perhaps more fruitful to look for possible basic similarities in the two hypotheses. It does seem that the eddy structure, at least in the region of the law of the wall, cannot be considered in isolation from the effects of the wall and the wall effects appear to be stronger than hitherto assumed.

There is, on the other hand, reason to question the assumption that the eddy length scale is proportional to the distance from the wall in axisymmetric flows. It is difficult to believe, for example that the eddy structure on cylinders of, say, 5 mm and 50 mm diameters should be the same at, say 2 mm from the wall even if the thickness of the turbulent boundary layer is the same in both cases. For example, the velocity induced by one ring vortex on another in the two cases will be different even if the strengths of the two vortices is the same in both cases. For this reason, the search for an alternate formulation for the "mixing length" is justified and the final law of the wall in Ref. 3 can be obtained if the mixing length $l$ is taken in the form

$$l = Ka(r/a)^{1/2}\ln(r/a) \tag{4}$$

The use of the stress variation in deriving Eq. (7) in Ref. 1 seems to have brought the results of Refs. 1 and 3 closer. Experimental justification for the assumption of the stress variation, however, is still lacking but all indirect evidences suggest that the form obtained in the sublayer persists in the region of the law of the wall also.

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